Exercice n°6 printemps maths :

Soit f, $\forall x > \frac{1}{\pi}$, $f(x) = \sqrt{\frac{\ln^2\left(\frac{x+1}{x}\right)}{\sin^3\left(\frac{1}{x}\right)}}$

1. DL à f, ordre $\frac{1}{x^{3/2}}$ en $+\infty$:

$\ln\left(\frac{1+x}{x}\right) = \ln\left(1+\frac{1}{x}\right)$. V

$\ln\left(1+\frac{1}{x}\right) \underset{x \to +\infty}{=} \frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} + o\left(\frac{1}{x^3}\right)$ oui

$\sin\left(\frac{1}{x}\right) \underset{x \to +\infty}{=} \frac{1}{x} - \frac{1}{6x^3} + o\left(\frac{1}{x^3}\right)$ V

Donc $f(x) \underset{x \to +\infty}{=} \left(\frac{\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} + o\left(\frac{1}{x^3}\right)}{\left(\frac{1}{x} - \frac{1}{6x^3} + o\left(\frac{1}{x^3}\right)\right)^{3/2}}\right)^{\cancel{1/2}}$ Non !

On factorise par le terme prépondérant

$\underset{x \to +\infty}{=} \left(\frac{1/x - \frac{1}{2x^2} + \frac{1}{3x^3} + o\left(\frac{1}{x^3}\right)}{\left(\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} + o\left(\frac{1}{x^3}\right)\right) + \frac{1}{2x^2} - \frac{1}{2x^3}}\right)^{1/2}$

OK à reprendre

$$\underset{x \mapsto +\infty}{=} \left(\frac{1}{1 + \frac{1/2x^2 - 1/2x^3}{1/x - 1/2x^2 + \frac{1}{3x^3} + o\left(\frac{1}{x^3}\right)}} \right)^{1/2}.$$

$$\underset{x \mapsto +\infty}{=} \left(1 - \frac{1/2x^2 - 1/2x^3}{1/x - \frac{1}{2x^2} + \frac{1}{3x^3} + o\left(\frac{1}{x^3}\right)} \right)^{1/2}$$

$$\underset{x \mapsto +\infty}{=} 1 - \frac{1}{2} \frac{1/2x^2 - 1/2x^3}{1/x - \frac{1}{2x^2} + \frac{1}{3x^3} + o\left(\frac{1}{x^3}\right)}$$

$$\underset{x \mapsto +\infty}{=} 1 - \frac{x^3 - x^2}{x^4 - \frac{1}{2}x^3 + \frac{1}{3}x^2 + o(x^2)}$$