Exercice Trigonométrie

Soit $\alpha \in [0;\pi]$ tel que $\cos(\alpha) = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

1. $\cos(2\alpha) = 2\cos^2(\alpha) - 1$ ✓

$$= 2 \times \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2 - 1 \quad ✓$$

$$= 2 \times \frac{6 - 2\sqrt{6}\,\sqrt{2} + 2}{16} - 1 \quad ✓$$

$$= 2 \times \frac{8 - 4\sqrt{3}}{16} - 1 \quad ✓$$

$$= \frac{16 - 8\sqrt{3}}{16} - 1 \quad ✓$$

$$= 1 - \frac{\sqrt{3}}{2} - 1$$

$$\boxed{\cos(2\alpha) = -\frac{\sqrt{3}}{2}}$$ oui

2. $\cos(2\alpha) = \dfrac{-\sqrt{3}}{2} \Leftrightarrow 2\alpha = \dfrac{5\pi}{6}$

Et pourquoi pas $-\dfrac{5\pi}{6}$? ou $\dfrac{5\pi}{6} + 2\pi$

A mieux justifier

$$\Leftrightarrow \boxed{\alpha = \frac{5\pi}{12}}$$

3. (Soit (E) : $\cos^2(\alpha) + \sin^2(\alpha) = 1$)

On a :

((E) $\Leftrightarrow$) $\sin^2(\alpha) = 1 - \cos^2(\alpha)$ ✓

$$\Leftrightarrow \cancel{2\sin^2(\alpha)} = 1 - \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2$$

$$\Leftrightarrow \cancel{2\sin^2(\alpha)} = \frac{4^2 - (\sqrt{6}-\sqrt{2})^2}{4^2} \quad ✓$$

$\Leftrightarrow \sin^2(\alpha) = \dfrac{4^2 - (6 - 2\sqrt{6}\sqrt{2} + 2)}{4^2}$ ✓

$\Leftrightarrow \cancel{\sin^2(\alpha)} = \dfrac{16 - 6 + 2\sqrt{6}\sqrt{2} - 2}{4^2}$ ✓

$\Leftrightarrow \cancel{\sin^2(\alpha)} = \dfrac{8 + 2\sqrt{12}}{4^2}$ ✓

$\Leftrightarrow \cancel{\sin^2(\alpha)} = \dfrac{8 + 4\sqrt{3}}{4^2}$ ✓

A justifier !!

$\Leftrightarrow \boxed{\sin(\alpha) = \dfrac{\sqrt{6} + \sqrt{2}}{4}}$

4. Soit (I) : $(\sqrt{6}+\sqrt{2})\cos(x) + (\sqrt{6}-\sqrt{2})\sin(x) \geq 2$ dans $\mathbb{R}$

$(I) \Leftrightarrow \dfrac{\sqrt{6}+\sqrt{2}}{4}\cos(x) + \dfrac{\sqrt{6}-\sqrt{2}}{4}\sin(x) \geq \dfrac{1}{2}$ ✓

$\Leftrightarrow \sin(\alpha)\cos(x) + \cos(\alpha)\sin(x) \geq \dfrac{1}{2}$ oui

$\Leftrightarrow \sin(\alpha + x) \geq \dfrac{1}{2}$ ✓

$\Leftrightarrow \sin(\alpha + x) \geq \sin\left(\dfrac{\pi}{6}\right)$ ✓

$\Rightarrow$

$\Leftrightarrow \dfrac{5\pi}{6} + 2k\pi \geq \alpha + x \geq \dfrac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

$\Rightarrow$

$\Leftrightarrow \dfrac{5\pi}{12} + 2k\pi \geq x \geq -\dfrac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$ car...

$\boxed{\mathcal{S} = \bigcup_{k\in\mathbb{Z}} \left\{ \left[\dfrac{5\pi}{12} + 2k\pi \,;\, \dfrac{-\pi}{4} + 2k\pi \right] \right\}}$

Bon travail.