TD0
Révisions de calculs

Exercice 1 - *Solution*. Simplifier les expressions suivantes :

1. $A = \frac{2}{3} - \frac{5}{12} + \frac{1}{9} - \frac{5}{6}$.
2. $B = \frac{\frac{\frac{2}{3}+2}{\frac{3}{4}-\frac{5}{6}} - \frac{1}{5}}{\frac{1}{\frac{2}{3}-\frac{3}{2}}}$.
3. $C = \frac{2^5 \times 25 \times 3^{-4} \times 36}{3^8 \times 15 \times 100}$.
4. $D = \frac{3\sqrt{72}}{2\sqrt{162}}$.
5. $E = \frac{\sqrt{3}-1}{\sqrt{3}+1}$.
6. $F = \sqrt{\frac{17}{18} + \frac{1}{3} + \frac{1}{2}}$
7. $G = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$.
8. $H = \left(\sqrt{2} + 5\sqrt{3}\right)\left(2 - \sqrt{3}\right)$.

Exercice 2 - *Solution*. Soit $x \in \mathbb{R}$. Développer les expressions suivantes.

1. $A = \left(x^2 - 2\right)^2$.
2. $B = (3x+1)^4$.
3. $C = (3x+2)^7$.
4. $D = \left(2x^2 - x + 1\right)^2$.
5. $E = \left(x^2 + 2x + 2\right)^3$.
6. $F = (x + y + z + t)^2$.

Exercice 3 - *Solution*. Soit $x \in \mathbb{R}$. Factoriser les expressions suivantes.

1. $A = 2x^2 - 12x + 18$.
2. $B = 4x^2 - 16$.
3. $C = (2x-6)(x+2) - (x+1)(x-3) + 2x(3-x)$.
4. $D = (2x+1)^3 + (2x+1)^2 + 2x + 1$.
5. $E = (x+1)^2 - 4x$.
6. $F = x^3 + 6x^2 + 14x + 12$.
7. $G = x^4 - 10x^3 + 35x^2 - 50x + 24$.
8. $H = x^4 - 2x^2 - 3$.
9. $I = x^3 - 1$.
10. $J = x^3 + 1$.
11. $K = x^4 + 1$.

Exercice 4 - *Solution*. Soit $a \in \mathbb{R}$. Simplifier les calculs suivants.

1. $A = \sqrt[3]{5^{12}}$.
2. $B = \sqrt[4]{27}\sqrt[4]{3}$.
3. $C = \sqrt[5]{a^3\sqrt[3]{a}}$.
4. $D = \sqrt[4]{8}\sqrt[4]{2}$.
5. $E = \sqrt[3]{\frac{1}{3}}\sqrt[3]{\frac{1}{9}}$.
6. $F = \sqrt{12}\sqrt{3}$.
7. $G = \sqrt[3]{2}\sqrt[3]{4}$.
8. $H = \sqrt[8]{81}\sqrt[8]{27}\sqrt[8]{3}$.
9. $I = \sqrt[14]{4^7}$.
10. $J = \sqrt{\sqrt{16}}$.
11. $K = \sqrt[7]{\sqrt{7^7}}$.
12. $L = \sqrt[3]{\sqrt{3^6}}$.

Exercice 5 - *Solution*. Soit $x \in \mathbb{R}$, simplifier les expressions suivantes :

1. $A = \mathrm{e}^x \mathrm{e}^{-x}$.
2. $B = \mathrm{e}^x + 3\mathrm{e}^x$.
3. $C = \left(\mathrm{e}^x\right)^3 \mathrm{e}^{-2x}$.
4. $D = \frac{\mathrm{e}^{3x}}{(\mathrm{e}^x)^2}$.
5. $E = \mathrm{e}^{3x+2}\mathrm{e}^{1-2x}$.
6. $F = \frac{\mathrm{e}^{2x+1}}{\frac{1}{\mathrm{e}^{-2x}}}$.
7. $G = \frac{\mathrm{e}^{3x-1}}{\mathrm{e}^{2-x}}$.
8. $H = \sqrt{3\mathrm{e}^{-x} + 6\mathrm{e}^{-x}}$.
9. $I = \sqrt{\frac{2\mathrm{e}^{3x+1}}{\mathrm{e}^{2x-1}}}$.

Exercice 6 - *Solution*. Exprimer en fonction de $\ln(2)$ et $\ln(3)$ les expressions suivantes :

1. $A = \frac{1}{2}\ln(16)$.
2. $B = \ln\left(\frac{1}{2}\right)$.
3. $C = \ln(36) - 2\ln(3)$.
4. $D = 2\ln\left(\frac{\sqrt{2}}{\sqrt{3}}\right)$.
5. $E = \ln(21) + 2\ln(14) - 3\ln(0,875)$.

Exercice 7 - *Solution*. Pour chacune des fractions suivantes, décrire l'ensemble des valeurs possibles dans $\mathbb{R}$ des paramètres puis rendre le dénominateur rationnel.

1. $A = \frac{1}{\sqrt{x+1}+\sqrt{x-1}}$.
2. $B = \frac{1}{\sqrt{a}+\sqrt{b}+\sqrt{a+b}}$.
3. $C = \frac{1}{\sqrt[3]{2}-1}$.
4. $D = \frac{1}{\sqrt[3]{2}+2}$.
5. $E = \frac{1}{\sqrt[4]{2}-1}$.
6. $F = \frac{1}{\sqrt[3]{x+1}-\sqrt[3]{x-1}}$.

Exercice 8 - *Solution*. Pour chacune des fonctions, déterminer son domaine de dérivabilité et calculer sa dérivée.

1. $f_1 : x \mapsto (4-3x)^3$.
2. $f_2 : x \mapsto (2x-1)^2(4-3x)^3$.
3. $f_3 : x \mapsto \frac{3}{(x^2+1)^2}$.
4. $f_4 : x \mapsto \frac{(3x-2)^3}{(1-2x)^2}$.
5. $f_5 : x \mapsto \frac{\sqrt{x}}{\left(3+2\sqrt{x}\right)^2}$.
6. $f_6 : x \mapsto \frac{x\left(\sqrt{x}+1\right)}{(2x^2+1)^3}$.
7. $f_7 : x \mapsto \cos^2(x)$.
8. $f_8 : x \mapsto (1+\tan(x))^2$.
9. $f_9 : x \mapsto \frac{1}{(2-\cos(3x))^2}$.
10. $f_{10} : x \mapsto \tan^2\left(3x + \frac{\pi}{3}\right)$.
11. $f_{11} : x \mapsto \sqrt{4x^2 - 3x - 1}$.
12. $f_{12} : x \mapsto \sqrt{\sin(x)}$.
13. $f_{13} : x \mapsto \frac{\mathrm{e}^{\frac{x-1}{x+2}} - 1}{3\mathrm{e}^x + 2}$.
14. $f_{14} : x \mapsto \frac{\mathrm{e}^{\frac{1}{x}}}{\sqrt{2}\mathrm{e}^x + 1}$.
15. $f_{15} : x \mapsto \ln(\ln(x))$.
16. $f_{16} : x \mapsto \ln\left(\frac{1-x}{3-2x}\right)$.

Exercice 9 - *Solution*. Calculer les limites suivantes :

1. $\lim\limits_{\substack{x\to 1\\ x<1}} \frac{2-x}{x^2-1}$.
2. $\lim\limits_{x\to+\infty} \frac{\sqrt{x}+2}{x+1}$.
3. $\lim\limits_{\substack{x\to 0\\ x\neq 0}} \frac{\mathrm{e}^{2x}-1}{x}$.
4. $\lim\limits_{x\to+\infty} \frac{\mathrm{e}^x - x}{x^2+3}$.
5. $\lim\limits_{x\to+\infty} \frac{\mathrm{e}^x + 5}{2\mathrm{e}^x + \mathrm{e}^{-x}}$.
6. $\lim\limits_{\substack{x\to 0\\ x>0}} \frac{-1}{\ln(x)-1}$.
7. $\lim\limits_{x\to+\infty} \frac{x\ln\left(x^3\right)}{x^2+1}$.
8. $\lim\limits_{x\to+\infty} \frac{\ln(x)}{x} + \frac{x^2-1}{2x}$.

Réponses

Solution de l'exercice 1 - *Enoncé*.

1. $A = -\frac{17}{36}$.
2. $B = \frac{161}{6}$.
3. $C = \frac{2^5}{3^{11}\times 5} = \frac{32}{885735}$.
4. $D = 1$.
5. $E = 2 - \sqrt{3}$.
6. $F = \frac{4}{3}$.
7. $G = 2$.
8. $H = 2\sqrt{2} + 10\sqrt{3} - \sqrt{6} - 15$.

Solution de l'exercice 2 - *Enoncé*.

1. $A = x^4 - 4x^2 + 2$.
2. $B = 81x^4 + 108x^3 + 54x^2 + 12x + 1$.
3. $C = 2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128$.
4. $D = 4x^4 - 4x^3 + 5x^2 - 2x + 1$.
5. $E = x^6 + 6x^5 + 18x^4 + 32x^3 + 36x^2 + 24x + 8$.
6. $F = x^2 + y^2 + z^2 + t^2 + 2xy + 2xz + 2xt + 2yz + 2yt + 2zt$.

Solution de l'exercice 3 - *Enoncé*.

1. $A = 2\left(x-3\right)^2$.
2. $B = 4\left(x-2\right)\left(x+2\right)$.
3. $C = -\left(x-3\right)^2$.
4. $D = \left(2x+1\right)\left(4x^2+6x+3\right)$.
5. $E = \left(x-1\right)^2$.
6. $F = \left(x+2\right)\left(x^2+4x+6\right)$.
7. $G = \left(x-1\right)\left(x-2\right)\left(x-3\right)\left(x-4\right)$.
8. $H = \left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)\left(x^2+1\right)$.
9. $I = \left(x-1\right)\left(x^2+x+1\right)$.
10. $J = \left(x+1\right)\left(x^2-x+1\right)$.
11. $K = \left(x^2-\sqrt{2}x+1\right)\left(x^2+\sqrt{2}x+1\right)$.

Solution de l'exercice 4 - *Enoncé*.

1. $A = 625$.
2. $B = 3$.
3. $C = \sqrt[3]{a^2}$.
4. $D = 2$.
5. $E = \frac{1}{3}$.
6. $F = 6$.
7. $G = 2$.
8. $H = 3$.
9. $I = 2$.
10. $J = 2$.
11. $K = \sqrt{7}$.
12. $L = 3$.

Solution de l'exercice 5 - *Enoncé*.

1. $A = 1$.
2. $B = 4\,\mathrm{e}^{x}$.
3. $C = \mathrm{e}^{x}$.
4. $D = \mathrm{e}^{x}$.
5. $E = \mathrm{e}^{x+3}$.
6. $F = \mathrm{e}$.
7. $G = \mathrm{e}^{4x-3}$.
8. $H = 3\,\mathrm{e}^{-\frac{x}{2}}$.
9. $I = \sqrt{2}\,\mathrm{e}^{\frac{x}{2}+1}$.

Solution de l'exercice 6 - *Enoncé*.

1. $A = 2\ln\left(2\right)$.
2. $B = -\ln\left(2\right)$.
3. $C = 2\ln\left(2\right)$.
4. $D = \ln\left(2\right) - \ln\left(3\right)$.
5. $E = 11\ln\left(2\right) + \ln\left(3\right)$.

Solution de l'exercice 7 - *Enoncé*.

1. $\forall x \geqslant 1,\ A = \frac{\sqrt{x+1}-\sqrt{x-1}}{2}$.
2. $B = \begin{cases} \frac{a\sqrt{b}+b\sqrt{a}-\sqrt{(a+b)ab}}{2ab} & \text{si } (a,b)\in\left(\mathbb{R}_+^*\right)^2 \\ \frac{\sqrt{b}}{2b} & \text{si } a=0 \text{ et } b>0 \\ \frac{\sqrt{a}}{2a} & \text{si } a>0 \text{ et } b=0 \end{cases}\cdot$
3. $C = \sqrt[3]{4} + \sqrt[3]{2} + 1$.
4. $D = \frac{\sqrt[3]{4}-2\sqrt[3]{2}+4}{10}$.
5. $E = \sqrt[4]{8} + \sqrt{2} + \sqrt[4]{2} + 1$.
6. $\forall x\in\mathbb{R},\ F = \frac{\sqrt[3]{(x+1)^2}+\sqrt[3]{(x^2-1)}+\sqrt[3]{(x-1)^2}}{2}$.

Solution de l'exercice 8 - *Enoncé*.

1. $\mathscr{D}_{f_1} = \mathbb{R},\ f_1' : x \mapsto -9\left(4-3x\right)^2$.
2. $\mathscr{D}_{f_2} = \mathbb{R},\ f_2' : x \mapsto 5\left(4-3x\right)^2\left(2x-1\right)\left(5-6x\right)$.
3. $\mathscr{D}_{f_3} = \mathbb{R},\ f_3' : x \mapsto -\frac{12x}{(x^2+1)^3}$.
4. $\mathscr{D}_{f_4} = \mathbb{R}\setminus\left\{\frac{1}{2}\right\},\ f_4' : x \mapsto \frac{(3x-2)^2(1-6x)}{(1-2x)^3}$.
5. $\mathscr{D}_{f_5} = \mathbb{R}_+^*,\ f_5' : x \mapsto \frac{3-2\sqrt{x}}{2\sqrt{x}\left(3+2\sqrt{x}\right)^3}$.
6. $\mathscr{D}_{f_6} = \mathbb{R}_+^*,\ f_6' : x \mapsto \frac{3\sqrt{x}\left(x^2-4x+\frac{1}{2}\right)-12x+1}{(2x^2+1)^4}$.
7. $\mathscr{D}_{f_7} = \mathbb{R},\ f_7' : x \mapsto -2\sin(x)\cos(x)$.
8. $\mathscr{D}_{f_8} = \mathbb{R}\setminus\left\{\frac{\pi}{2}+k\pi \;\middle|\; k\in\mathbb{Z}\right\},\ f_8' : x \mapsto 2\left(1+\tan^2(x)\right)\left(1+\tan(x)\right)$.
9. $\mathscr{D}_{f_9} = \mathbb{R},\ f_9' : x \mapsto -\frac{6\sin(3x)}{(2-\cos(3x))^3}$.
10. $\mathscr{D}_{f_{10}} = \mathbb{R}\setminus\left\{\frac{\pi}{18}+k\frac{\pi}{3} \;\middle|\; k\in\mathbb{Z}\right\},\ f_{10} : x \mapsto 6\left(1+\tan^2\left(3x+\frac{\pi}{3}\right)\right)\tan\left(3x+\frac{\pi}{3}\right)$.
11. $\mathscr{D}_{f_{11}} = \left]-\infty;-\frac{1}{4}\right[\cup\left]1;+\infty\right[,\ f_{11} : x \mapsto \frac{8x-3}{2\sqrt{4x^2-3x-1}}$.
12. $\mathscr{D}_{f_{12}} = \underset{\bigcup}{k\in\mathbb{Z}}\left]2k\pi;(2k+1)\pi\right[,\ f_{12} : x \mapsto \frac{\cos(x)}{2\sqrt{sin(x)}}$.
13. $\mathscr{D}_{f_{13}} = \mathbb{R}\setminus\{-2\},\ f_{13} : x \mapsto 3\frac{\mathrm{e}^{\frac{x-1}{x+2}}(3\,\mathrm{e}^x+2)-(x+2)^2\left(\mathrm{e}^{\frac{x-1}{x+2}}-1\right)\mathrm{e}^x}{(3\,\mathrm{e}^x+2)^2}$.

14. $\mathscr{D}_{f_{14}} = \mathbb{R}^*$, $f_{14} : x \mapsto -\frac{e^{\frac{1}{x}}\left(e^x\left(x^2+2\right)+1\right)}{x^2(2e^x+1)^{3/2}}$.
15. $\mathscr{D}_{f_{15}} =]1;+\infty[$, $f_{15} : x \mapsto \frac{1}{x\ln(x)}$.
16. $\mathscr{D}_{f_{16}} =]-\infty;1[\cup]\frac{3}{2};+\infty[$, $f_{16} : x \mapsto \frac{1}{x-1} - \frac{2}{2x-3}$.

Solution de l'exercice 9 - *Enoncé*.

1. $\lim\limits_{\substack{x\to 1\\ x<1}} \frac{2-x}{x^2-1} = -\infty$.
2. $\lim\limits_{x\to+\infty} \frac{\sqrt{x}+2}{x+1} = 0$.
3. $\lim\limits_{\substack{x\to 0\\ x\neq 0}} \frac{e^{2x}-1}{x} = (\exp(2x))'_{|x=0} = 2$.
4. $\lim\limits_{x\to+\infty} \frac{e^x-x}{x^2+3} = +\infty$.
5. $\lim\limits_{x\to+\infty} \frac{e^x+5}{2e^x+e^{-x}} = \frac{1}{2}$.
6. $\lim\limits_{\substack{x\to 0\\ x>0}} \frac{-1}{\ln(x)-1} = 0$.
7. $\lim\limits_{x\to+\infty} \frac{x\ln\left(x^3\right)}{x^2+1} = 0$.
8. $\lim\limits_{x\to+\infty} \frac{\ln(x)}{x} + \frac{x^2-1}{2x} = +\infty$.