

TD0 Révisions de calculs

Exercice 1 - Solution. Simplifier les expressions suivantes :

$$1. A = \frac{2}{3} - \frac{5}{12} + \frac{1}{9} - \frac{5}{6}.$$

$$2. B = \frac{\frac{2}{3} + 2}{\frac{4}{3} - \frac{5}{6}} - \frac{1}{\frac{1}{3} - \frac{3}{2}}.$$

$$3. C = \frac{2^5 \times 25 \times 3^{-4} \times 36}{3^8 \times 15 \times 100}.$$

$$4. D = \frac{3\sqrt{72}}{2\sqrt{162}}.$$

$$5. E = \frac{\sqrt{3}-1}{\sqrt{3}+1}.$$

$$6. F = \sqrt{\frac{17}{18} + \frac{1}{3} + \frac{1}{2}}$$

$$7. G = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}.$$

$$8. H = (\sqrt{2} + 5\sqrt{3})(2 - \sqrt{3}).$$

Exercice 2 - Solution. Soit $x \in \mathbb{R}$. Développer les expressions suivantes.

$$1. A = (x^2 - 2)^2.$$

$$2. B = (3x + 1)^4.$$

$$3. C = (3x + 2)^7.$$

$$4. D = (2x^2 - x + 1)^2.$$

$$5. E = (x^2 + 2x + 2)^3.$$

$$6. F = (x + y + z + t)^2.$$

Exercice 3 - Solution. Soit $x \in \mathbb{R}$. Factoriser les expressions suivantes.

$$1. A = 2x^2 - 12x + 18.$$

$$2. B = 4x^2 - 16.$$

$$3. C = (2x - 6)(x + 2) - (x + 1)(x - 3) + 2x(3 - x).$$

$$4. D = (2x + 1)^3 + (2x + 1)^2 + 2x + 1.$$

$$5. E = (x + 1)^2 - 4x.$$

$$6. F = x^3 + 6x^2 + 14x + 12.$$

$$7. G = x^4 - 10x^3 + 35x^2 - 50x + 24.$$

$$8. H = x^4 - 2x^2 - 3.$$

$$9. I = x^3 - 1.$$

$$10. J = x^3 + 1.$$

$$11. K = x^4 + 1.$$

Exercice 4 - Solution. Soit $a \in \mathbb{R}$. Simplifier les calculs suivants.

$$1. A = \sqrt[3]{5^{12}}.$$

$$2. B = \sqrt[4]{27} \sqrt[4]{3}.$$

$$3. C = \sqrt[5]{a^3} \sqrt[3]{a}.$$

$$4. D = \sqrt[4]{8} \sqrt[4]{2}.$$

$$5. E = \sqrt[3]{\frac{1}{3}} \sqrt[3]{\frac{1}{9}}.$$

$$6. F = \sqrt{12} \sqrt{3}.$$

$$7. G = \sqrt[3]{2} \sqrt[3]{4}.$$

$$8. H = \sqrt[8]{81} \sqrt[8]{27} \sqrt[8]{3}.$$

$$9. I = \sqrt[14]{4^7}.$$

$$10. J = \sqrt{\sqrt{16}}.$$

$$11. K = \sqrt[7]{\sqrt{7^7}}.$$

$$12. L = \sqrt[3]{\sqrt{3^6}}.$$

Exercice 5 - Solution. Soit $x \in \mathbb{R}$, simplifier les expressions suivantes :

$$1. A = e^x e^{-x}.$$

$$2. B = e^x + 3e^x.$$

$$3. C = (e^x)^3 e^{-2x}.$$

$$4. D = \frac{e^{3x}}{(e^x)^2}.$$

$$5. E = e^{3x+2} e^{1-2x}.$$

$$6. F = \frac{e^{2x+1}}{e^{\frac{1}{2-2x}}}.$$

$$7. G = \frac{e^{3x-1}}{e^{2-x}}.$$

$$8. H = \sqrt{3e^{-x} + 6e^{-x}}.$$

$$9. I = \sqrt{\frac{2e^{3x+1}}{e^{2x-1}}}.$$

Exercice 6 - Solution. Exprimer en fonction de $\ln(2)$ et $\ln(3)$ les expressions suivantes :

$$1. A = \frac{1}{2} \ln(16).$$

$$2. B = \ln\left(\frac{1}{2}\right).$$

$$3. C = \ln(36) - 2 \ln(3).$$

$$4. D = 2 \ln\left(\frac{\sqrt{2}}{\sqrt{3}}\right).$$

$$5. E = \ln(21) + 2 \ln(14) - 3 \ln(0,875).$$

Exercice 7 - Solution. Pour chacune des fractions suivantes, décrire l'ensemble des valeurs possibles dans \mathbb{R} des paramètres puis rendre le dénominateur rationnel.

$$1. A = \frac{1}{\sqrt{x+1} + \sqrt{x-1}}.$$

$$2. B = \frac{1}{\sqrt{a} + \sqrt{b} + \sqrt{a+b}}.$$

$$3. C = \frac{1}{\sqrt[3]{2}-1}.$$

$$4. D = \frac{1}{\sqrt[3]{2}+2}.$$

$$5. E = \frac{1}{\sqrt[4]{2}-1}.$$

$$6. F = \frac{1}{\sqrt[3]{x+1} - \sqrt[3]{x-1}}.$$

Exercice 8 - Solution. Pour chacune des fonctions, déterminer son domaine de dérivabilité et calculer sa dérivée.

$$1. f_1 : x \mapsto (4 - 3x)^3.$$

$$2. f_2 : x \mapsto (2x - 1)^2 (4 - 3x)^3.$$

$$3. f_3 : x \mapsto \frac{3}{(x^2 + 1)^2}.$$

$$4. f_4 : x \mapsto \frac{(3x - 2)^3}{(1 - 2x)^2}.$$

$$5. f_5 : x \mapsto \frac{\sqrt{x}}{(3 + 2\sqrt{x})^2}.$$

$$6. f_6 : x \mapsto \frac{x(\sqrt{x} + 1)}{(2x^2 + 1)^3}.$$

$$7. f_7 : x \mapsto \cos^2(x).$$

$$8. f_8 : x \mapsto (1 + \tan(x))^2.$$

$$9. f_9 : x \mapsto \frac{1}{(2 - \cos(3x))^2}.$$

$$10. f_{10} : x \mapsto \tan^2(3x + \frac{\pi}{3}).$$

$$11. f_{11} : x \mapsto \sqrt{4x^2 - 3x - 1}.$$

$$12. f_{12} : x \mapsto \sqrt{\sin(x)}.$$

$$13. f_{13} : x \mapsto \frac{\frac{x-1}{x+2} - 1}{3e^x + 2}.$$

$$14. f_{14} : x \mapsto \frac{e^{\frac{1}{x}}}{\sqrt{2e^x + 1}}.$$

$$15. f_{15} : x \mapsto \ln(\ln(x)).$$

$$16. f_{16} : x \mapsto \ln\left(\frac{1-x}{3-2x}\right).$$

Exercice 9 - Solution. Calculer les limites suivantes :

$$1. \lim_{\substack{x \rightarrow 1 \\ x < 1}} \frac{2-x}{x^2 - 1}.$$

$$2. \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + 2}{x + 1}.$$

$$3. \lim_{\substack{x \rightarrow 0 \\ x \neq 0}} \frac{e^{2x} - 1}{x}.$$

$$4. \lim_{x \rightarrow +\infty} \frac{e^x - x}{x^2 + 3}.$$

$$5. \lim_{x \rightarrow +\infty} \frac{e^x + 5}{2e^x + e^{-x}}.$$

$$6. \lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{-1}{\ln(x) - 1}.$$

$$7. \lim_{x \rightarrow +\infty} \frac{x \ln(x^3)}{x^2 + 1}.$$

$$8. \lim_{x \rightarrow +\infty} \frac{\ln(x)}{x} + \frac{x^2 - 1}{2x}.$$

Réponses

Solution de l'exercice 1 - *Enoncé*.

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|---------------------------|---|--|
| 1. $A = -\frac{17}{36}$. | 2. $B = \frac{161}{6}$. | 3. $C = \frac{2^5}{3^{11} \times 5} = \frac{32}{885735}$. |
| 4. $D = 1$. | 5. $E = 2 - \sqrt{3}$. | 6. $F = \frac{4}{3}$. |
| 7. $G = 2$. | 8. $H = 2\sqrt{2} + 10\sqrt{3} - \sqrt{6} - 15$. | |

Solution de l'exercice 2 - *Enoncé*.

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|--|---|
| 1. $A = x^4 - 4x^2 + 2$. | 2. $B = 81x^4 + 108x^3 + 54x^2 + 12x + 1$. |
| 3. $C = 2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128$. | |
| 4. $D = 4x^4 - 4x^3 + 5x^2 - 2x + 1$. | |
| 5. $E = x^6 + 6x^5 + 18x^4 + 32x^3 + 36x^2 + 24x + 8$. | |
| 6. $F = x^2 + y^2 + z^2 + t^2 + 2xy + 2xz + 2xt + 2yz + 2yt + 2zt$. | |

Solution de l'exercice 3 - *Enoncé*.

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|--|--|
| 1. $A = 2(x-3)^2$. | 2. $B = 4(x-2)(x+2)$. |
| 3. $C = -(x-3)^2$. | 4. $D = (2x+1)(4x^2+6x+3)$. |
| 5. $E = (x-1)^2$. | 6. $F = (x+2)(x^2+4x+6)$. |
| 7. $G = (x-1)(x-2)(x-3)(x-4)$. | 8. $H = (x-\sqrt{3})(x+\sqrt{3})(x^2+1)$. |
| 9. $I = (x-1)(x^2+x+1)$. | 10. $J = (x+1)(x^2-x+1)$. |
| 11. $K = (x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)$. | |

Solution de l'exercice 4 - *Enoncé*.

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|----------------|------------------------|--------------------------|
| 1. $A = 625$. | 2. $B = 3$. | 3. $C = \sqrt[3]{a^2}$. |
| 4. $D = 2$. | 5. $E = \frac{1}{3}$. | 6. $F = 6$. |
| 7. $G = 2$. | 8. $H = 3$. | 9. $I = 2$. |
| 10. $J = 2$. | 11. $K = \sqrt{7}$. | 12. $L = 3$. |

Solution de l'exercice 5 - *Enoncé*.

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|---------------------|------------------------------|--------------------------------------|
| 1. $A = 1$. | 2. $B = 4e^x$. | 3. $C = e^x$. |
| 4. $D = e^x$. | 5. $E = e^{x+3}$. | 6. $F = e$. |
| 7. $G = e^{4x-3}$. | 8. $H = 3e^{-\frac{x}{2}}$. | 9. $I = \sqrt{2}e^{\frac{x}{2}+1}$. |

Solution de l'exercice 6 - *Enoncé*.

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|----------------------------|-------------------------------|---------------------|
| 1. $A = 2 \ln(2)$. | 2. $B = -\ln(2)$. | 3. $C = 2 \ln(2)$. |
| 4. $D = \ln(2) - \ln(3)$. | 5. $E = 11 \ln(2) + \ln(3)$. | |

Solution de l'exercice 7 - *Enoncé*.

1. $\forall x \geqslant 1, A = \frac{\sqrt{x+1}-\sqrt{x-1}}{2}$.
2. $B = \begin{cases} \frac{a\sqrt{b}+b\sqrt{a}-\sqrt{(a+b)ab}}{2ab} & \text{si } (a,b) \in (\mathbb{R}_+^*)^2 \\ \frac{\sqrt{b}}{2b} & \text{si } a=0 \text{ et } b>0 \\ \frac{\sqrt{a}}{2a} & \text{si } a>0 \text{ et } b=0 \end{cases}$.
3. $C = \sqrt[3]{4} + \sqrt[3]{2} + 1$.
4. $D = \frac{\sqrt[3]{4}-2\sqrt[3]{2}+4}{10}$.
5. $E = \sqrt[4]{8} + \sqrt{2} + \sqrt[4]{2} + 1$.
6. $\forall x \in \mathbb{R}, F = \frac{\sqrt[3]{(x+1)^2} + \sqrt[3]{(x^2-1)} + \sqrt[3]{(x-1)^2}}{2}$.

Solution de l'exercice 8 - *Enoncé*.

1. $\mathcal{D}_{f_1} = \mathbb{R}, f'_1 : x \mapsto -9(4-3x)^2$.
2. $\mathcal{D}_{f_2} = \mathbb{R}, f'_2 : x \mapsto 5(4-3x)^2(2x-1)(5-6x)$.
3. $\mathcal{D}_{f_3} = \mathbb{R}, f'_3 : x \mapsto -\frac{12x}{(x^2+1)^3}$.
4. $\mathcal{D}_{f_4} = \mathbb{R} \setminus \left\{ \frac{1}{2} \right\}, f'_4 : x \mapsto \frac{(3x-2)^2(1-6x)}{(1-2x)^3}$.
5. $\mathcal{D}_{f_5} = \mathbb{R}_+^*, f'_5 : x \mapsto \frac{3-2\sqrt{x}}{2\sqrt{x}(3+2\sqrt{x})^3}$.
6. $\mathcal{D}_{f_6} = \mathbb{R}_+^*, f'_6 : x \mapsto \frac{3\sqrt{x}(x^2-4x+\frac{1}{2})-12x+1}{(2x^2+1)^4}$.
7. $\mathcal{D}_{f_7} = \mathbb{R}, f'_7 : x \mapsto -2 \sin(x) \cos(x)$.
8. $\mathcal{D}_{f_8} = \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k\pi \mid k \in \mathbb{Z} \right\}, f'_8 : x \mapsto 2(1 + \tan^2(x))(1 + \tan(x))$.
9. $\mathcal{D}_{f_9} = \mathbb{R}, f'_9 : x \mapsto -\frac{6 \sin(3x)}{(2-\cos(3x))^3}$.
10. $\mathcal{D}_{f_{10}} = \mathbb{R} \setminus \left\{ \frac{\pi}{18} + k\frac{\pi}{3} \mid k \in \mathbb{Z} \right\}, f_{10} : x \mapsto 6(1 + \tan^2(3x + \frac{\pi}{3})) \tan(3x + \frac{\pi}{3})$.
11. $\mathcal{D}_{f_{11}} =]-\infty; -\frac{1}{4}[\cup]1; +\infty[, f_{11} : x \mapsto \frac{8x-3}{2\sqrt{4x^2-3x-1}}$.
12. $\mathcal{D}_{f_{12}} = \bigcup_{k \in \mathbb{Z}} [2k\pi; (2k+1)\pi], f_{12} : x \mapsto \frac{\cos(x)}{2\sqrt{\sin(x)}}$.
13. $\mathcal{D}_{f_{13}} = \mathbb{R} \setminus \{-2\}, f_{13} : x \mapsto 3 \frac{\frac{x-1}{e^{x+2}}(3e^x+2)-(x+2)^2 \left(\frac{x-1}{e^{x+2}} - 1 \right) e^x}{(3e^x+2)^2}$.

14. $\mathcal{D}_{f_{14}} = \mathbb{R}^*, f_{14} : x \mapsto -\frac{e^{\frac{1}{x}}(e^x(x^2+2)+1)}{x^2(2e^x+1)^{3/2}}.$

15. $\mathcal{D}_{f_{15}} =]1; +\infty[, f_{15} : x \mapsto \frac{1}{x \ln(x)}.$

16. $\mathcal{D}_{f_{16}} =]-\infty; 1[\cup]\frac{3}{2}; +\infty[, f_{16} : x \mapsto \frac{1}{x-1} - \frac{2}{2x-3}.$

Solution de l'exercice 9 - Enoncé.

1. $\lim_{\substack{x \rightarrow 1 \\ x < 1}} \frac{2-x}{x^2-1} = -\infty.$

2. $\lim_{x \rightarrow +\infty} \frac{\sqrt{x}+2}{x+1} = 0.$

3. $\lim_{\substack{x \rightarrow 0 \\ x \neq 0}} \frac{e^{2x}-1}{x} = (\exp(2x))'_{|x=0} = 2.$

4. $\lim_{x \rightarrow +\infty} \frac{e^x-x}{x^2+3} = +\infty.$

5. $\lim_{x \rightarrow +\infty} \frac{e^x+5}{2e^x+e^{-x}} = \frac{1}{2}.$

6. $\lim_{\substack{x \rightarrow 0 \\ x > 0}} \frac{-1}{\ln(x)-1} = 0.$

7. $\lim_{x \rightarrow +\infty} \frac{x \ln(x^3)}{x^2+1} = 0.$

8. $\lim_{x \rightarrow +\infty} \frac{\ln(x)}{x} + \frac{x^2-1}{2x} = +\infty.$