Exercice 2 :

1. $N^2 =$

~~$\begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$~~

$$\begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix}$$ ✓

$$N^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ ✓

$N^n = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ lorsque n est ≥ 3. Oui

$A^2 = \begin{pmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$ $A^3 = \begin{pmatrix} 4 & 0 & 0 \\ 12 & 4 & 0 \\ 3 & 4 & 4 \end{pmatrix}$

$$\begin{pmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 4 & 0 & 0 \\ 12 & 4 & 0 \\ 3 & 4 & 4 \end{pmatrix} \qquad \begin{pmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 8 & 0 & 0 \\ 36 & 8 & 0 \\ 18 & 12 & 8 \end{pmatrix}$$

$A = N + D$ où $D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$A^n = (N+D)^n$
On suppose
$N^2 = N$
$N^2 \neq 0$
$N^3 = 0$

A bien revoir

$D^n = \begin{pmatrix} 2^n & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 2^n \end{pmatrix}$

Exercice 1.

DL$_3(0)$ de $g : x \mapsto \dfrac{x^2 - \sin(x)}{\operatorname{sh}(x)}$

$\sin(x) \underset{x\to 0}{=} x - \dfrac{x^3}{6} + \dfrac{x^5}{120} + o(x^3)$ ✓

$x^2 - \sin(x) \underset{x\to 0}{=} x^2 - \left(x - \dfrac{x^3}{6} + \dfrac{x^5}{120} + o(x^3)\right)$ ✓

$\underset{x\to 0}{=} x^2 - x + \dfrac{x^3}{6} + \dfrac{x^5}{120} + o(x^3)$ ✓

$\operatorname{sh}(x) \underset{x\to 0}{=} x + \dfrac{x^3}{6} + o(x^3)$ ✓

$\dfrac{1}{\operatorname{sh}(x)} \underset{x\to 0}{=} \dfrac{1}{x + \frac{x^3}{6}} + o(x^3)$ ✓ !

$\underset{x\to 0}{=} \dfrac{1}{x} + \dfrac{6}{x^3} + o(x^3)$

$\dfrac{1}{a+b} \neq \dfrac{1}{a} + \dfrac{1}{b}$

A revoir,

$\dfrac{x^2 - \sin(x)}{\operatorname{sh}(x)} \underset{x\to 0}{=} \left(x^2 - x + \dfrac{x^3}{6} + o(x^3)\right) \times \left(\dfrac{1}{x} + \dfrac{6}{x^3} + o(x^3)\right)$

$\underset{x\to 0}{=} \left(x + \dfrac{x^2}{6} + o(x^3)\right) \times \left(\dfrac{6}{x^3}\right)$

$\underset{x\to 0}{=} \dfrac{6x}{x^3} + \dfrac{6x^2}{6x^3} + o(x^3)$

$\underset{x\to 0}{=} \dfrac{6}{x^2} + \dfrac{1}{x} + o(x^3)$