Exercice Noël 08

$DL_3(0)$ de $f : x \longmapsto \frac{x^2 - \sin(x)}{sh(x)}$

$\sin(x) \xrightarrow[x \to 0]{} 0$ ✓

$sh(x) \xrightarrow[x \to 0]{} 0$ ✓

$\sin(x) \underset{x \to 0}{=} x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{7!} + o(x^7)$ ✓

$sh(x) \underset{x \to 0}{=} x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5)$ ✓

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$f(x) \underset{x \to 0}{=} \frac{x^2 - x + \frac{x^3}{6} - \cancel{\frac{x^5}{120}} + o(x^3)}{x - \frac{x^3}{6} + \cancel{\frac{x^5}{120}} + o(x^3)} = \frac{x^2}{x - \frac{x^3}{6} + o(x^3)} - 1$

A calculer !!

2. $A = \begin{pmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$; $N = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$N^2 = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix}$ ✓

$N^3 \begin{pmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = O_3$ ✓

Et donc N^n ?

$Q = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ $A = Q + N$

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Rédaction ?????

Hypothèse ? Nom de la formule ?

$A^n = (Q+N)^n = \sum_{k=0}^{n} \binom{n}{k} N^k Q^{n-k} = Q^n + nNQ^{n-1} + \binom{n}{2} N^2 Q^{n-2}$