Exo DL

Soit $f : x \longmapsto \arctan\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

On pose $u = x^2 \underset{x \to 0}{\longrightarrow} 0$

oui

$$\sqrt{1+u} \underset{u \to 0}{=} 1 + \frac{u}{2} + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2} u^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6} u^3 + o(u^3)$$

$$\underset{u \to 0}{=} 1 + \frac{u}{2} - \frac{1}{8} u^2 + \frac{1}{16} u^3 + o(u^3).$$

De plus :

$$\sqrt{1+x^2} \underset{x \to 0}{=} 1 + \frac{x^2}{2} - \frac{1}{8}(x^2)^2 + \frac{1}{16}(x^3)^2 + o((x^3)^2)$$

$$\underset{x \to 0}{=} 1 + \frac{1}{2} x^2 - \frac{1}{8} x^4 + \frac{1}{16} x^6 + o(x^6)$$

Ainsi :

$$\frac{\sqrt{1+x^2}-1}{x} \underset{x \to 0}{=} 1 - 1 + \frac{\frac{1}{2} x^2}{x} - \frac{\frac{1}{8} x^4}{x} + \frac{\frac{1}{16} x^6}{x} + \frac{o(x^6)}{x}$$

$$\underset{x \to 0}{=} \frac{1}{2} x - \frac{1}{8} x^3 + \frac{1}{16} x^5 + o(x^5) = V(x)$$

$$(V(x))^2 \underset{x \to 0}{=} \frac{x^2}{4} - \frac{x^4}{16} + o(x^5) \underset{x \to 0}{=} \frac{x^2}{4} - \frac{2x^4}{16} + o(x^5)$$

$$- \frac{x^4}{16} + o(x^5)$$

$$+ o(x^5)$$

$$V(x)^3 \underset{x\to0}{=} \left(\frac{x^2}{4} - \frac{x^4}{8} + o(x^5)\right)\left(\frac{x}{2} - \frac{x^3}{8} + \frac{x^5}{16} + o(x^5)\right)$$ ✓

$$\underset{x\to0}{=} \frac{x^3}{8} - \frac{x^5}{32} + o(x^5)$$
$$- \frac{x^5}{16} + o(x^5)$$ ✓
$$+ o(x^5)$$

$$\underset{x\to0}{=} \frac{x^3}{8} - \frac{3x^5}{32} + o(x^5)$$ ouf ok

$$\cancel{V(x)^4 \underset{x\to0}{=} \left(\frac{x^3}{8} - \frac{3x^5}{32} + o(x^5)\right)\left(\frac{x}{2} - \frac{x^3}{8} + \frac{x^5}{16} + o(x^5)\right)}$$

$$\cancel{\underset{x\to0}{=}}$$

inutile car DL de l'ordre est uniquement sur des puissances impaires. OK

$$V(x)^5 \underset{x\to0}{=} \left(\frac{x^3}{8} - \frac{3x^5}{32} + o(x^5)\right)\left(\frac{x^2}{4} - \frac{x^4}{8} + o(x^5)\right)$$

$$\underset{x\to0}{=} \frac{x^5}{32} + o(x^5)$$ ✓

Aussi, $o(u(x)^5) \underset{x\to0}{=} o\left(\frac{x^5}{32} + o(x^5)\right) \underset{x\to0}{=} o(x^5)$ ✓

Pour finir :

$\arctan(u) \underset{u\to0}{=} u - \frac{u^3}{3} + \frac{u^5}{5} + o(u^5)$ Oui

Pour $u(x) = V(x)$ on a : $V(x) \xrightarrow[x\to0]{} 0$ ⚠

$$f(x) \underset{x \to 0}{=} \arctan(v(x))$$

$$\underset{x \to 0}{=} \frac{x}{2} - \frac{x^3}{8} + \frac{x^5}{16} + o(x^5)$$

$$- \frac{x^3}{24} + \frac{x^5}{32} + o(x^5)$$

$$+ \frac{x^5}{160} + o(x^5)$$

$$+ o(x^5)$$

TB

$$\underset{x \to 0}{=} \frac{x}{2} - \frac{4x^3}{24} + \frac{x^5}{160} + \frac{10x^5}{160} + \frac{5x^5}{160} + o(x^5)$$

$$\underset{x \to 0}{=} \frac{x}{2} - \frac{4x^3}{24} + \frac{16}{160} x^5 + o(x^5)$$

$$\boxed{f(x) \underset{x \to 0}{=} \frac{x}{2} - \frac{x^3}{6} + \frac{x^5}{10} + o(x^5)}$$

TB !

Exo SL

Soit $m \in \mathbb{R}$

$$\begin{cases} x + y + mz = m \\ x + my - z = 1 \\ x + y - z = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + mz = m \\ y(m-1) - z(1+m) = \cancel{1} - m \\ -z(1+m) = 1 - m \end{cases}$$

Opérations ??

$$\Leftrightarrow \begin{cases} x + y + mz = m \\ y(m-1) = 0 \\ -z(1+m) = \cancel{1} - m \end{cases}$$

Opération ?

$$\Leftrightarrow \begin{cases} x + y + mz = m \\ y(m-1) = 0 \\ -z \cancel{-zm} - 1 + m = 0 \end{cases}$$

A reprendre

$$\Leftrightarrow \begin{cases} x + y + mz = m \\ ym - y = 0 \\ \cancel{z = -1} \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + m = m \\ y = 0 \text{ ou } m = 1 \\ z = -1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y = 2m \\ y = 0 \text{ ou } m = 1 \\ z = -1 \end{cases}$$