

Exercice Automne 10.

$$(z_1, z_2, z_3) \in \mathbb{C}^3.$$

$$\text{Donc : } z_1 z_2 z_3 = 4\sqrt{2}(-1+i). \checkmark$$

$$\text{Arg}(z_1) \in \left[\frac{\pi}{2}, \pi \right] \checkmark$$

$$2|z_1| = |z_2| \quad 4|z_1| = |z_3|.$$

$$\text{Arg}(z_1) + \frac{\pi}{4} = \text{Arg}(z_2) \quad \checkmark$$

$$\text{Arg}(z_1) + \frac{\pi}{2} = \text{Arg}(z_3). \quad \checkmark$$

$$\begin{aligned} |z_1 z_2 z_3| &= |z_1| \times 2|z_1| \times 4|z_1| \\ &= 8|z_1|^3. \quad \checkmark \end{aligned}$$

$$\begin{aligned} |4\sqrt{2}(-1+i)| &= \sqrt{(-4\sqrt{2})^2 + (4\sqrt{2})^2} \\ &= \sqrt{32 + 32} \\ &= 8. \quad \text{Bien} \end{aligned}$$

$$\begin{aligned} \text{Donc } 8|z_1|^3 &= 8 \quad \checkmark \quad \text{Donc } |z_2| = 2 \\ |z_1|^3 &= 1 \\ |z_1| &= 1. \quad \text{Bien.} \quad \text{et } |z_3| = 4. \end{aligned}$$

$$\begin{aligned} \text{Arg}(z_1) + \text{Arg}(z_2) + \text{Arg}(z_3) &= \text{Arg}(z_1) \\ + \left(\text{Arg}(z_1) + \frac{\pi}{4} \right) + \left(\text{Arg}(z_1) + \frac{\pi}{2} \right) &= 3\text{Arg}(z_1) \\ + \frac{3\pi}{4}. \quad \checkmark \end{aligned}$$

L'arg($z_1 z_2 z_3$) n'écrit aussi.

not $k \in \mathbb{Z}$, ($i \in \mathbb{C}$).

$$\text{Arg}(\sqrt[4]{2}(-1+i)) = \text{Arg}(-1+i) = \frac{3\pi}{4} + 2k\pi \quad \checkmark \quad \text{OK}$$

$$3 \text{Arg}(z_1) + \frac{3\pi}{4} = \frac{3\pi}{4} [2\pi] \quad \checkmark$$

$$\Rightarrow 3 \text{Arg}(z_1) = 0 [2\pi]$$

$$\Rightarrow \text{Arg}(z_1) = 0 \in \frac{2\pi}{3} \quad \text{Bien.}$$

D'après l'énoncé $\text{Arg}(z_1) \in [\frac{\pi}{2}, \pi) \quad \checkmark$

$$\text{Arg}(z_1) = \frac{2\pi}{3} \quad \text{Bien}$$

$$\text{Arg}(z_2) = \text{Arg}(z_1) + \frac{\pi}{4}$$

$$= \frac{2\pi}{3} + \frac{\pi}{4}$$

$$= \frac{11\pi}{12} \quad \checkmark$$

$$\text{Arg}(z_3) = \text{Arg}(z_1) + \frac{\pi}{2}$$

$$= \frac{2\pi}{3} + \frac{\pi}{2}$$

$$= \frac{7\pi}{6} \quad \checkmark$$

Ainsi

$$z_1 = 1 e^{i \frac{2\pi}{3}} = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$$

$$= -\frac{1}{2} + i \frac{\sqrt{3}}{2}$$

$$z_2 = 2 e^{i \frac{11\pi}{12}} \checkmark = 2 \left(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12} \right)$$

$$= \frac{-\sqrt{6} + \sqrt{2}}{2} + i \frac{\sqrt{6} - \sqrt{2}}{2}$$

$$z_3 = 4 e^{i \frac{7\pi}{6}} = 4 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right)$$

$$= -2\sqrt{3} - 2i$$

Encadrer la conclusion.